International Journal of Applied Mathematics and Theoretical Physics
2019; 5(3): 52-57
http://www.sciencepublishinggroup.com/j/ijamtp
doi: 10.11648/j.ijamtp.20190503.11
ISSN: 2575-5919 (Print); ISSN: 2575-5927 (Online)

Quantum Wave Function Based on String Theory for Frictional Medium to Obtain Collision Probability, Energy Operator and Schrodinger Equation

Asma Mohamed Elhussin

Qilwah College of Science and Arts, Department of Physics, Albaha University, Al Baha, Saudi Arabia

Email address:
Asmmoh85@gmail.com

To cite this article:
Asma Mohamed Elhussin. Quantum Wave Function Based on String Theory for Frictional Medium to Obtain Collision Probability, Energy Operator and Schrodinger Equation. *International Journal of Applied Mathematics and Theoretical Physics.* Vol. 5, No. 3, 2019, pp. 52-57. doi: 10.11648/j.ijamtp.20190503.11

Received: March 24, 2019; **Accepted**: May 21, 2019; **Published**: August 12, 2019

Abstract: Schrodinger equation suffer from being not sensitive to the mechanical properties as well as the electric and magnetic properties of matter. This set back can be cured by starting the derivation using the function sensitive to these parameters. Treating particle as vibrating strings a useful expression for the velocity was found using the equation of motion. Then the relation of current density with a velocity and electric field intensity was utilized to obtain the electric field intensity in a frictional medium. Using the analogy of the electric field and quantum wave function, the wave function was obtained and found to give the conventional expression for the collision probability with relaxation time twice the classical one. Another approach was tackled by obtaining a useful expression of the total energy of strings for resistive collisional medium. This expression utilizes the wave function of quantum particle in a frictional medium to obtain collision probability formula. Fortunately this latter approach gives a relaxation time equal to the classical one. The same wave function is used to find Hamiltonian operator for the both steady state and perturbed state by friction. Fortunately both Hamiltonians satisfy hermiticty condition. The hermiticty condition for the perturbed states however needs splitting the Hamiltonian into unpertured and perturbed part.. The perturbed term satisfies uncertainty principle. The energy expression for the resistive medium resembles that of Einstein and RLC circuits. Schrodinger equation for the frictional medium was also found, where it reduces to the ordinary one when friction disappear.

Keywords: String, Collision Probability, Relaxation Time, Energy Operator, Schrodinger Eqauation, Hermiticty

1. Introduction

Matters are formed from aggregate of atoms to be in a solid, liquid or gaseous form. To understand the behavior of matter it is thus important to know the nature of atoms. According to the laws of quantum mechanics, the atom consists of a central nucleus surrounded by electrons moving in specific energy levels [1, 2].

The behaver of atom is described by Schrodinger equation for relativity slowly moving particles compared to the speed of light. Very fast particles can be described by Klien-Gorden and Dirac equations.

Schrodinger equation succeeded in explaining a wide variety of atomic phenomena, like atoms spectra, Zeeman Effect and hyperfine interaction [3, 4].

Despite these successes, quantum laws are far from describing some superconductor behavior like pressure and isotope effect [5, 6].

Describing and unifing gravity with other forces under the umbrella of quantum laws needs intensive research and a lot of work [7. 8].

These drawbacks of Schrodinger equation may be related to fact that it is based on the expression of the electric field intensity and the wave function in free space. Such functions are not sensitive to the properties of the medium like friction, electric permittivity and magnetic permeability.

To cure these setbacks many attempts were made to construct models that account for the effect of bulk matter properties like conductivity, friction and relaxation time [9-11]. Different attempts were also made to the bulk matter or elementary particles behavior [12, 13]. Attempts were also made to solve Schrodinger equation so as to describe some bulk matter or atomic behavior [14-16]. This model uses classical neutron second law and classical string theory to find a useful expression for the wave function which was used to obtain the conventional statistical collision probability relation. This is done in section (2). Sections (3) and section (4) are devoted for discussion and conclusion.

2. String Model for Collision

The equation of motion of a particle of mass (m) moving with velocity (v) in medium of coefficient γ takes the form.

$$m\frac{dv}{dt} = -\gamma v = -\frac{m}{\tau}v \tag{1}$$

Therefore.

$$\int_{v0}^{v}\frac{dv}{v} = -\int_{0}^{t}\frac{dt}{\tau}$$

$$ln\frac{v}{v_0} = -\frac{t}{\tau}$$

Thus the solution describes time decaying velocity.

$$v = v_0\, e^{-\frac{t}{\tau}} \tag{2}$$

But the current density is giving by.

$$J = nev = nev_0 e^{-\frac{t}{\tau}} = \sigma E \tag{3}$$

This requires

$$E = E_0\, e^{-\frac{t}{\tau}} \tag{4}$$

Bearing in mind the analogy between E and ψ, one.

$$\Psi = \Psi_0\, e^{-\frac{t}{\tau}} \tag{5}$$

Thus the probability of collision due the friction is given by.

$$P_r = |\Psi|^2 = |\Psi_0|^2\, e^{-\frac{t}{2\tau}} = P_{cr}\, e^{-\frac{t}{2\tau}} \tag{6}$$

But the ordinary collision probability function satisfies.

$$P = P_0\, e^{-\frac{t}{t_0}} \tag{7}$$

Thus this model resembles Lutfi model where.

$$\Psi = \Psi_0\, e^{-\frac{t}{2\tau}}$$

And

$$\tau_0 = 2t \tag{8}$$

The form of E in the presence of electric field can be found by using the relation between current density J and electric field intensity E which is given by.

$$J = nev = \sigma E$$

$$v = \frac{\sigma}{ne}E = C_i E \tag{9}$$

Thus one can suggest the solutions.

$$v = v_0\, e^{-i\omega t}f(t) = v_i f(t) \tag{10}$$

$$E = E_0\, e^{-i\omega t}f(t) = E_i f(t) \tag{11}$$

Where $C_i = \frac{v_0}{E_0}$

This gives.

$$v = \frac{v_i}{E_i}E = \frac{v_0}{E_0}E \tag{12}$$

$$E = \frac{E_i}{v_i}v = \frac{E_0}{v_0}v \tag{13}$$

Consider now the equation of motion of the electron in the resistive medium under electric field effect, i e.

$$m\frac{dv}{dt} = eE - \gamma v = -(e\frac{v_0}{E_0} - \gamma)v \tag{14}$$

Rearranging yields.

$$\int_{vi}^{v}\frac{dv}{v} = -(e\frac{v_0}{mE_0} - \frac{\gamma}{m})\int_{0}^{t}dt$$

$$ln\frac{v}{vi} = (e\frac{v_0}{mE_0} - \frac{\gamma}{m})t$$

$$v = vi\, e^{-(e\frac{v_0}{mE_0} - \frac{\gamma}{m})t} \tag{15}$$

Equation (10) assumes that.

$$v = v_0\, e^{-i\omega t}f(t) = v_i f(t)$$

Thus equation (15) reads.

$$v = v_0\, e^{-i\omega t}\, e^{-(e\frac{v_0}{mE_0} - \frac{\gamma}{m})t} \tag{16}$$

$$v = v_m\, e^{-i\omega t} \tag{17}$$

Where v_m stands for decaying amplitude.

But.

$$J = nev = \sigma E$$

$$J = nev = n\,e\,v_0\, e^{-i\omega t} = \sigma\, E_0 e^{-i\omega t} \tag{18}$$

Thus conductivity is given by.

$$\sigma = \frac{nev_0}{E_0} \tag{19}$$

Hence.

$$\frac{E_0}{v_0} = \frac{ne}{\sigma} \tag{20}$$

As a result equation (17) reads.

$$v = v_0\, e^{-i\omega t}\, e^{-(\frac{ne^2}{\sigma} - \gamma)t} \tag{21}$$

In the presence of magnetic and electric field the equation of motion is given by.

$$m\frac{dv}{dt} = e\,E - B_o ev - \gamma v \tag{22}$$

Where B_0 is a constant magnetic field, therefore.

$$m\frac{dv}{dt} = e\left(\frac{E_0}{v_0}\right).v - B_o ev - \gamma v = -C_1 v \tag{23}$$

Where.

$$C_1 = \gamma - e\left(\frac{E_0}{v_0}\right) + B_o e = \gamma - \frac{ne^2}{\sigma} + B_o e \tag{24}$$

Rearranging (23) requires.

$$\int_{vi}^{v}\frac{dv}{v} = -C_1\int_0^t dt$$

$$ln\frac{v}{vi} = -C_1 t$$

$$v = vi\,e^{-C_1 t} \tag{25}$$

In view of (10), (11) and (13).

$$E = E_0\,e^{-i\omega t}\,e^{-C_1 t} \tag{26}$$

The complete travelling wave solution is given by.

$$E = E_0\,e^{i(kx-\omega t)}\,e^{-C_1 t} \tag{27}$$

Again the analogy between E and ψ requires.

$$\Psi = \Psi_0\,e^{\frac{i}{h}(Px-Et)}\,e^{-C_1 t} \tag{28}$$

An alternative way can also be used to find the wave function for resistive medium.

For harmonic oscillator the displacement is given by.

$$x = x_0\,e^{+i\omega t}$$

$$v = \frac{dx}{dt} = i\omega x_0\,e^{-i\omega t} = v_0\,e^{-i\omega t} = i\omega x$$

$$x_0 = \frac{iv_0}{\omega} \tag{29}$$

For no potential field.

$$m\frac{dv}{dt} = -\gamma v - \nabla v = -\gamma v - \frac{\partial V}{\partial x}$$

$$m\frac{dx}{dt}\frac{dv}{dx} = mv\frac{dv}{dx}$$

$$\frac{dV}{dx} + \frac{d(\frac{1}{2}mv^2)}{dx} = -\gamma v$$

$$\int dV + \int dT = -\gamma\int vdx + C = -\frac{\gamma}{i\omega}\int vdv + C$$

$$T + V = \frac{i\gamma}{w}\left(\frac{1}{2}v^2\right) + C$$

For harmonic oscillator the potential is given by.

$$V = -\int F.dx = k\int xdx = m\omega^2\left|\frac{x^2}{2}\right| = \frac{m\omega^2}{2}\left|\frac{v}{i\omega}\right|^2 = \frac{1}{2}mv^2 = T$$

$$T + V - \frac{i\gamma}{w}\left(\frac{1}{2}v^2\right) = C \tag{30}$$

Thus the total energy is given by.

$$E = 2T - \frac{i\gamma}{mw}T = C \tag{31}$$

Where for non-frictional system, the energy of the oscillator is given by.

$$E = E_o + E_f\ ;\ E_o = T + V = 2T$$

Where.

$$T = V$$

For harmonic oscillator.

For frictional medium the energy is given by.

$$E = E_0 + E_f \tag{32}$$

Where.

$$E_f = -\frac{i\gamma}{mw}T = -\frac{i\gamma h}{mwh}\frac{E_0}{2} = -\frac{i\gamma h}{mE_0}\frac{E_0}{2} = \frac{-i\gamma h}{2m} = -\frac{imh}{2m\tau} = -\frac{ih}{2\tau} \tag{33}$$

Thus according to equations (32) and (33) the wave function for free particle is given by.

$$\Psi = A\,e^{\frac{i}{h}(Px-Et)} = A\,e^{\frac{i}{h}(Px-E_0t)}e^{\frac{-i}{h}(\frac{-i\gamma h)t}{m}} = \Psi_m\,e^{\frac{i}{h}(Px-E_0t)}$$

$$\Psi_m = A\,e^{-t\frac{\gamma}{2m}} = A\,exp(\frac{-t}{2\tau}) \tag{34}$$

Thus the collision probability is given by.

$$P_r = |\Psi^2| = \Psi\overline{\Psi} = A^2 e^{-\frac{t}{\tau}} = P_0 e^{-\frac{t}{\tau}} \tag{35}$$

3. Schrodinger Equation for Frictional Medium

Schrodinger equation is based on wave particle duality, which require the wave function to be in the form.

$$\Psi = Ae^{\frac{i}{h}(px-Et)} \tag{36}$$

This expression describes the wave function in free space. Thus one expects it not to feel the properties of the medium like friction, electric permittivity and magnetic permeability.

This makes Schrodinger equation in the conventional traditional form unable to solve the problems with the bulk matter. This cause so called many body problems, such problem was solved by introducing new terms adding to the potential describing different interactions like spin-spin interaction, spin-lattice interaction. These approaches make Schrodinger equation very complex and make its solution very tedius.

To make treatment with bulk matter simple, it is better to start with from wave function of particles in a frictional medium which is given by equation (34) to be in the form.

$$\Psi = A\, e^{\frac{i\hbar}{\hbar}(px-Et)} \quad (37)$$

The energy operator can be found by differentiating w. r. t to get.

$$\frac{\partial \Psi}{\partial t} = -\left(\frac{1}{2\tau} + \frac{i}{\hbar}\right)\Psi$$

$$-\frac{\hbar}{i}\frac{\partial}{\partial t}\Psi = \left(\frac{\hbar}{2\tau}i + E\right)\Psi$$

$$\frac{\hbar}{i}\frac{\partial \Psi}{\partial t} = \left(E + \frac{\hbar}{2\tau}i\right)\Psi \quad (38)$$

The energy operator can be defined by two different ways: according to equation (38) if the energy operator is that gives steady state energy, then the energy operator $\hat{H}$ is given by.

$$\hat{H}_s\Psi = i\hbar\left(\frac{\partial}{\partial t} - \frac{1}{2\tau}\right)\Psi = E\Psi \quad (39)$$

The steady energy operator satisfies.

$$\hat{H}_s = i\hbar\left(\frac{\partial}{\partial t} - \frac{1}{2\tau}\right) \quad (40)$$

i.e However if the energy operator is defined to be that gives the energy afte perturbation, then energy operator gives.

$$\hat{H}_p\Psi = i\hbar\frac{\partial}{\partial t}\Psi = \left(E + \frac{\hbar}{2\tau}i\right)\Psi \quad (41)$$

i.e the perturbing Hamiltonian is given by.

$$\hat{H}_p = i\hbar\frac{\partial}{\partial t} \quad (42)$$

This additional term standing for friction in (41) conforms with uncertainty principle where.

$$\widetilde{\Delta E}\widetilde{\Delta t} = \hbar \quad (43)$$

Following Zeeman Effect, if one level is splitted to two sublevels by an amount ΔE up and down, due to collision which causes friction.

The splitting between the two levels is thus given by.

$$\widetilde{\Delta E} = 2\Delta E \quad (44)$$

Thus when the particles take time τ in the excited state.

$$\Delta t = \tau \quad (45)$$

And equations (43), (44) and (45) gives.

$$(2\Delta E)\,\tau = \hbar$$

$$\Delta E = \frac{\hbar}{2\,\tau} \quad (46)$$

This gives the same expression for perturbation energy in equation (41).

The satisfaction of hermiticity condition for the steady and perturbed Hamiltonian can be examined also using equations (39-42).

For steady state Hamiltonian hermiticity requires.

$$\int \overline{\hat{H}_s\Psi}\,\Psi\, dr = \int \overline{\Psi}\,\hat{H}_s\Psi\, dr \quad (47)$$

Using equation (39) requires.

$$\int \overline{E}\,\overline{\Psi}\Psi\, dr = \int \overline{E}\,\hat{H}_s\Psi\, dr$$

$$\overline{E}\int \overline{\Psi}\Psi\, dr = E\int \overline{\Psi}\Psi\, dr$$

Thus.

$$\overline{E} = E \quad (48)$$

This conforms to the requirement of hermiticity.

The perturbation Hamiltonian hermiticity condition requires.

$$\int \overline{\hat{H}_P\Psi}\,\Psi\, dr = \int \overline{\Psi}\,\hat{H}_P\Psi\, dr$$

$$\int \overline{\left(E + \frac{\hbar}{2\tau}i\right)}\,\overline{\Psi}\Psi\, dr = \int \overline{\Psi}\left(E + \frac{\hbar}{2\tau}i\right)\Psi\, dr$$

$$\left(\overline{E} - \frac{\hbar}{2\tau}i\right)\int \overline{\Psi}\Psi\, dr = \left(E + \frac{\hbar}{2\tau}i\right)\int \overline{\Psi}\Psi\, dr$$

$$\overline{E} - \frac{\hbar}{2\tau}i = E + \frac{\hbar}{2\tau}i \quad (49)$$

But since E is real its follows that the frictional term vanishes where.

$$iE_f = \frac{i\hbar}{\tau} = 0 \; E_f = \frac{\hbar}{\tau} = 0 \quad (50)$$

Thus satisfy hermiticity condition, one requires.

$$\int \overline{\Psi(\hat{H} - i\hat{H}_f)}\,\Psi dr = \int \overline{\Psi}\,(\hat{H} + i\hat{H}_f)\Psi dr$$

Which gives.

$$E + iE_f = E + iE_f \quad (51)$$

Thus the perturbed Hamilton satisfies the hermiticity condition. Following the expression of energy of RCL circuit for alternating current, the perturbed energy is given by.

$$E_p^2 - E^2 + \frac{\hbar}{2\tau^2} \quad (52)$$

This resembles Einstein momentum-Energy relation.

$$E^2 = C^2P^2 + m_o^2C^4 \quad (53)$$

To find Schrodinger equation (37) is differentiated w. r. t coordinate to get.

$$\frac{\partial\Psi}{\partial x} = \frac{i}{\hbar}P\Psi$$

$$\frac{\hbar}{i}\nabla\Psi = \frac{\hbar}{i}\frac{\partial\Psi}{\partial x} = P\Psi$$

$$-\hbar^2\nabla^2\Psi = P^2\Psi \quad (54)$$

Using newton energy-momentum relation.

$$E = \frac{P^2}{2m} + V \quad (55)$$

After multiplying (51) by Ψ and inserting equations (38) and (54) in equation (55) where.

$$E\Psi = \frac{P^2}{2m}\Psi + V\Psi \quad (56)$$

One gets.

$$i\hbar\frac{\partial\Psi}{\partial t} - \frac{i\hbar}{2\tau}\Psi = V\Psi - \frac{\hbar^2}{2m}\nabla^2\Psi \quad (57)$$

This represents Schrodinger equation for frictional medium.

4. Discussion

Equation (2) uses the equation of motion of particle in frictional medium to obtain exponentially decaying velocity. The relation of current density J with the velocity and electric field in (3) to find exponentially decaying electric field in equation (4). This is used to find The wave function which is too exponentially decaying as shown in equation (5). This equation is used to find a useful expression of the collision probability (7) which is typical to the conventional one after redefining the relaxation time.

The equation of motion in the presence friction and magnetic field in equation was solved. To obtain travelling exponentially decaying wave function in equation (28). The decay depends on friction, conductivity as well as the magnetic field. Treating the particles as strings the collision probability obtained by our model in equation (35) is typical to the ordinary one.

The energy for frictional medium in equation (52) resembles that of Einstein and RLC circuit. This analogy is clear when comparing with Einstein energy momentum equation (53). The wave function of frictional medium is obtained in equation (37). It consists of additional time decaying termas equation (38) indicates. This function is used to find the energy operator for steady state which gives frictionless energy as equation (39) and equation (40) shows. However for perturbed state the energy operator gives both steady and frictional energy [see equations (41) & (42). Using uncertainty relation the same expression of frictional energy isfound [equation (46)]. Both Hamiltonians satisfy hermiticity [equation (47) & (49)].

5. Conclusion

Bearing in the mind the electric field intensity and quantum wave function analogy, a useful wave function sensitive to the medium properties is obtained. Using the relation between the current density with both velocity and electric field intensity a useful expression of the wave function was found by using Newton second law for a particle affected by electric and magnetic field as well as friction. Assuming that the particles acting as strings an expression for the collision probability density typical to the conventional one is obtained. A useful expression for Hamiltonian for steady and perturbed term satisfying hemiticity is also found. The energy of resistive medium resembles that of Einstein and RLC circuit one. A useful Schrodinger equation for resistive medium is also found and reduces to the ordinary Schrodinger equation in the absence of friction.

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