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Traveling Wave Solutions of Nonlinear Evolution Equations Via the Modified Simple Equation Method

A. K. M. Kazi Sazzad Hossain[1, *], M. Ali Akbar[2]

[1]Department of Mathematics, Begum Rokeya University, Rangpur, Bangladesh
[2]Department of Applied Mathematics, University of Rajshahi, Rajshahi, Bangladesh

Email address:
kazi_bru@yahoo.com (A. K. M. K. S. Hossain), ali_math74@yahoo.com (M. A. Akbar)
[*]Corresponding author

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Abstract: Exact solutions of nonlinear evolution equations (NLEEs) are very crucial to realize the obscurity of many physical phenomena in mathematical science. The modified simple equation (MSE) method is especially effective and highly proficient mathematical instrument to obtaining exact traveling wave solutions to NLEEs arising in science, engineering and mathematical physics. Though the modified simple equation method effectively provides exact traveling wave solutions to nonlinear evolution equations for balance number less or equal 2 but the method did not applied to solve if the balance number is greater than 2. In this article, the MSE method is executed to construct exact traveling wave solutions of nonlinear evolution equations namely Kuramoto-Sivashinsky equation with balance number equal to 3. The obtained solutions are expressed in terms of exponential and trigonometric functions including solitary and periodic solutions. Moreover the procedure of this method reduces huge volume of calculations.

Keywords: Modified Simple Equation (MSE) Method, Kuramoto-Sivashinsky Equation, Nonlinear Evolution Equations (NLEEs), Exact Traveling Wave Solutions

1. Introduction

The nonlinear evolution equations (NLEEs) are now extensively used in order to express the obscurity and inner mechanism of many physical phenomena in various fields of applied mathematics, science and engineering. Since nonlinear process are one of the most challenges and cannot be control easily because of its nonlinear characteristics hence NLEEs appear in a wide range of scientific research in various fields, such as nuclear physics, high energy physics, solid state physics, plasma physics, biomechanics, fluid mechanics, fluid dynamics, gas dynamics, optical fibers, elasticity, biochemistry, chemical reactions, geochemistry, meteorology etc. As a result, it is very essential to explore for promote exact traveling solutions to NLEEs for better realization. Consequently diverse groups of researchers have been working vigorously to develop effective methods for obtaining close form or exact solutions to NLEEs. That's why, recently several methods have been establish to explore exact solution, such as the nonlinear transform method [1], the functional variable method [2], the homogeneous balance method [3, 4], the direct algebraic method [5], the rank analysis method [6], the Jacobi-elliptic function expansion method [7], the complex hyperbolic function method [8], the tanh-function method [9], the inverse scattering transform [10], the Exp-function method [11-13], the sine-cosine method [14], the first integration method [15], the auxiliary parameter method [16], the Painleve expansion method [17], the Adomian decomposition method [18], the generalized Riccati equation method [19], the Lie group symmetry method [20], the modified Exp-function method [21], the perturbation method [22], the $\exp(-\Phi(\eta))$ -expansion method [23-25], the (G'/G)-expansion method [26-28], the asymptotic method [29], the improve (G'/G)-expansion method [30], the modified simple equation method [31-34] etc. The recently developed modified simple equation method is getting popularity in use because of its straight forward calculation procedure but the method did not applied to solve if the balance number is greater than 2. To the best of our knowledge, till now only four or five articles are available in the literature concerning higher balance number (for balance number two) [32].

The objective of this article is to establish the modified simple equation method to construct fresh and more general exact traveling wave solutions to NLEEs namely Kuramoto-Sivashinsky equation with balance number equal to 3. The rest of the article is arranged as follows: In Section 2, modified simple equation method is discussed. In Section 3, the MSE method is applied to investigate the NLEEs indicated above. In Section 4, results are discussed and In Section 5 conclusions are provided.

2. Outline of the Modified Simple Equation Method

To explain the modified simple equation method, let us consider a nonlinear evolution equation in two independent variables x and t in the form:

$$P\left(u, u_t, u_x, u_{tt}, u_{xx}, u_{xt}, \dots\right) = 0, \tag{1}$$

where $u = u(x,t)$ is an unknown function and P is a polynomial of $u(x,t)$ and its partial derivatives wherein the highest order derivatives and nonlinear terms are involved and the subscripts are used for partial derivatives. The essential steps of this method are presented in the following:

Step 1: Initiating a compound variable ξ, we combine the real variables x and t:

$$u(x,t) = u(\xi), \xi = x \pm \omega\, t, \tag{2}$$

where ω is the speed of the traveling wave.

The traveling wave transformations (2) allow us in reducing Eq. (1) into an ODE for $u = u(\xi)$ in the form:

$$Q(\mathrm{u}, u', u'', u''' \dots) = 0, \tag{3}$$

where Q is a polynomial in $u(\xi)$ and its derivatives, the prime denotes the derivative with respect to ξ.

Step 2: Assume the solution of (3) can be expressed of the form:

$$u(\xi) = \sum_{i=0}^{N} a_i \left(\frac{\psi'(\xi)}{\psi(\xi)}\right)^i \tag{4}$$

where $a_i (i = 0, 1, 2, 3, \dots N)$ arearbitrary constants to be determined such that $a_N \neq 0$ and $\psi(\xi)$ is an unknown function to be evaluated later, such that $\psi'(\xi) \neq 0$. The characteristic and uniqueness of this method is that, $\psi(\xi)$ is not known function or not a solution of any uncomplicated equation, whereas in the Exp-function method, tanh-function method, (G'/G)-expansion method, sine-cosine method, Jacobi elliptic function method etc. the solution are proposed in terms of known function. Thus, it might be possible to obtain some new solution by this method.

Step 3: We determine the positive integer$N$arisesin (4) by balancing the highest order of linear and nonlinear terms appearing in (3).

Step 4: Compute the necessary derivatives $u', u'', \dots$ and insert Eq. (4) into (3) and then we account the function $\psi(\xi)$. The above procedure makes a polynomial in $(1/\psi(\xi))$. Equating the coefficients of same power of this polynomial to zero, yields a system of algebraic and differential equations that can be solved to get $a_i (i = 0, 1, 2, 3, \dots N)$ and $\psi(\xi)$. This completes the determination of solutions of Eq. (1).

3. Applications of the Method

In this section, the modified simple equation method (MSE) has been applied to examine the closed form solutions leading to solitary wave solutions to the Kuramoto-Sivashinsky equation. Let us consider the Kuramoto-Sivashinsky equation in the form [27]:

$$u_t = -u u_x - u_{xx} - u_{xxxx} \tag{5}$$

This equation has applications in water waves and one-dimensional evolution of small amplitude long waves in several problems arising in fluid dynamics.

The traveling wave transformations

$u(x,t) = u(\xi), \xi = x - \omega t$, where ω is a constant to be determined latter, reduce the Eq. (5) into an ODE of the form,

$$u^{(iv)} + u''' + u u' - \omega u' = 0 \tag{6}$$

Integrating (6) with respect to ξ twice and the setting the constant of integration to zero, we can obtain the following result

$$u''' + u' + \frac{u^2}{2} - \omega u = 0. \tag{7}$$

Balancing the order of nonlinear and linear terms u^2 and u''' respectively, we obtain $N = 3$

Therefore, the solution of Eq. (7) becomes,

$$u(\xi) = a_0 + a_1 \left(\frac{\psi\prime}{\psi}\right) + a_2 \left(\frac{\psi\prime}{\psi}\right)^2 + a_3 \left(\frac{\psi\prime}{\psi}\right)^3 \tag{8}$$

where a_0, a_1, a_2 and a_3 are constants, such that $a_3 \neq 0$ and $\psi(\xi)$ is an unknown function to be calculated. Substituting (8) and its derivatives into (7) and then equating the coefficients of $\psi^0, \psi^{-1}, \psi^{-2}, \psi^{-3}, \psi^{-4}, \psi^{-5}, \psi^{-6}$ to zero we achieveseven successive algebraic and differential equations,

$$\frac{a_0^2}{2} - \omega a_0 = 0 \tag{9}$$

$$-\omega a_1 \psi' + a_0 a_1 \psi' + a_1 \psi'' + a_1 \psi^{(iv)} = 0 \tag{10}$$

$$2a_2\psi'\psi'' + 2a_2\psi'\psi^{(iv)} - 4a_1\psi'\psi''' + a_0 a_2 \psi'^2 + 6a_2\psi''\psi''' - a_1^2\psi'^2 - 3a_1\psi''^2 - \omega a_2\psi'^2 - 3a_1\psi\psi'' + \frac{a_1^2}{2}\psi'^2 = 0 \tag{11}$$

$$12a_1\psi'^2\psi'' + 3a_3\psi'^2\psi^{(iv)} + 18a_3\psi'\psi''\psi''' + 6a_3\psi''^3 - \omega a_3\psi'^3 + 3a_3\psi'^2\psi'' + a_0 a_3\psi'^3 + a_1 a_2\psi'^3 - 2a_2\psi'^3 - 14a_2\psi'^2\psi''' - 24a_2\psi'\psi''^2 = 0 \tag{12}$$

$$-81a_3a_1\psi'^2\psi''^2 - 30a_3\psi'^3\psi''' - 3a_3\psi'^4 + a_1a_3\psi'^4 + 54a_2\psi'^3\psi'' - 6a_1\psi'^4 + \frac{a_2^2}{2}\psi'^4 = 0 \quad (13)$$

$$144a_3\psi'^4\psi'' - 24a_2\psi'^5 + a_2a_3\psi'^5 = 0 \quad (14)$$

$$-60a_3\psi'^6 + \frac{a_3^2}{2}\psi'^6 = 0 \quad (15)$$

From Eq. (9), it is found that $a_0 = 0$ and $a_0 = 2\omega$.
And from Eq. (15), we achieve $a_3 = 120$, since $a_3 \neq 0$.
From Eq. (14), it can be deduced that

$$\frac{\psi''}{\psi'} = \lambda \quad (16)$$

Where

$$\lambda = \frac{a_2(24 - a_3)}{144a_3} \quad (17)$$

Integrating (14) with respect to ξ, yields

$$\psi' = c_1 e^{\lambda\xi} \quad (18)$$

And

$$\psi = \frac{c_1 e^{\lambda\xi}}{\lambda} + c_2 \quad (19)$$

where c_1 and c_2 are arbitrary constants.

Case 1: When $a_0 = 0$ and $a_3 = 120$, solving Eq. (10)-(12) by using (17)-(19), provides $a_1 = \frac{720}{19}$, $a_2 = \frac{180}{19}\sqrt{209}$ and $\omega = -\frac{30}{361}\sqrt{209}$. Then by setting these values of a_0, a_1, a_2, a_3 and ω in (8) yields

$$u(\xi) = \frac{-\frac{60}{361}\sqrt{209}c_1{}^3 e^{3\lambda\xi/2} + \frac{540}{361}\sqrt{209}c_1{}^2c_2\lambda e^{\frac{\lambda\xi}{2}} - \frac{720}{361}\sqrt{209}c_1c_2{}^2\lambda^2 e^{-\lambda\xi/2}}{(c_1e^{\lambda\xi/2} + c_2\lambda e^{-\lambda\xi/2})^3} \quad (20)$$

where $\xi = (x - \omega t)$, and $\lambda = -\frac{\sqrt{209}}{19}$

Simplifying the exponential solution transformed to the trigonometric function in the close form solution of the equation (20) as

$$u(\xi) = \frac{-\frac{60}{361}\sqrt{209}c_1{}^3\left(\cos\left(\frac{3\lambda\xi}{2}\right) + i\sin\left(\frac{3\lambda\xi}{2}\right)\right) + A\cos\left(\frac{\lambda\xi}{2}\right) + iB\sin\left(\frac{\lambda\xi}{2}\right)}{\left((c_1 + c_2\lambda)\cos\left(\frac{\lambda\xi}{2}\right) + (c_1 - c_2\lambda)i\sin\left(\frac{\lambda\xi}{2}\right)\right)^3} \quad (21)$$

where $A = \frac{540}{361}\sqrt{209}c_1{}^2c_2\lambda - \frac{720}{361}\sqrt{209}c_1c_2{}^2\lambda^2$ and $B = \frac{540}{361}\sqrt{209}c_1{}^2c_2\lambda + \frac{720}{361}\sqrt{209}c_1c_2{}^2\lambda^2$. Since c_1 and c_2 are integration constants, one may randomly pick their values, Therefore, if we set $c_1 = 1$ and $c_2 = 1/\lambda$ in Eq. (21), we obtain the following closed form solution of the Kuramoto-Sivashinsky equation:

$$u(\xi) = -\frac{\sqrt{209}}{722}sec^2\left(\frac{\lambda\xi}{2}\right)\left[\left(15\cos\left(\frac{3\lambda\xi}{2}\right)sec\left(\frac{\lambda\xi}{2}\right) + 45\right) + i\left(15\sin\left(\frac{3\lambda\xi}{2}\right)sec\left(\frac{\lambda\xi}{2}\right) - 315tan\left(\frac{\lambda\xi}{2}\right)\right)\right] \quad (22)$$

where $\xi = (x - \omega t)$, and $\lambda = -\frac{\sqrt{209}}{19}$

Again choosing $c_1 = -1$ and $c_2 = 1/\lambda$ the closed form solution (22) turn as

$$u(\xi) = -\frac{\sqrt{209}}{722}cosec^2\left(\frac{\lambda\xi}{2}\right)\left[\left(315 - 15\sin\left(\frac{3\lambda\xi}{2}\right)cosec\left(\frac{\lambda\xi}{2}\right)\right) + i\left(15\cos\left(\frac{3\lambda\xi}{2}\right)cosec\left(\frac{\lambda\xi}{2}\right) + 45tan\left(\frac{\lambda\xi}{2}\right)\right)\right] \quad (23)$$

Case 2: When $a_0 = 2\omega$, $a_1 = \frac{720}{19}$, $a_2 = \frac{180}{19}\sqrt{209}$, $a_3 = 120$ and $\omega = \frac{30}{361}\sqrt{209}$, then (8) gives

$$u(\xi) = \frac{60}{361}\sqrt{209} + \frac{-\frac{60}{361}\sqrt{209}c_1{}^3 e^{3\lambda\xi/2} + \frac{540}{361}\sqrt{209}c_1{}^2c_2\lambda e^{\frac{\lambda\xi}{2}} - \frac{720}{361}\sqrt{209}c_1c_2{}^2\lambda^2 e^{-\lambda\xi/2}}{(c_1e^{\lambda\xi/2} + c_2\lambda e^{-\lambda\xi/2})^3} \quad (24)$$

where $\xi = (x - \omega t)$, and $\lambda = -\frac{\sqrt{209}}{19}$

Converting the exponential function into the trigonometric identity, the close form solution (24) becomes

$$u(\xi) = \frac{60}{361}\sqrt{209} + \frac{-\frac{60}{361}\sqrt{209}c_1{}^3\left(\cos\left(\frac{3\lambda\xi}{2}\right) + i\sin\left(\frac{3\lambda\xi}{2}\right)\right) + C\cos\left(\frac{\lambda\xi}{2}\right) + iD\sin\left(\frac{\lambda\xi}{2}\right)}{\left((c_1 + c_2\lambda)\cos\left(\frac{\lambda\xi}{2}\right) + (c_1 - c_2\lambda)i\sin\left(\frac{\lambda\xi}{2}\right)\right)^3} \quad (25)$$

Where $C = \frac{540}{361}\sqrt{209}c_1{}^2c_2\lambda - \frac{720}{361}\sqrt{209}c_1c_2{}^2\lambda^2$ and $D = \frac{540}{361}\sqrt{209}c_1{}^2c_2\lambda + \frac{720}{361}\sqrt{209}c_1c_2{}^2\lambda^2$ Setting $c_1 = 1$ and $c_2 = 1/\lambda$ in Eq. (25), we obtain the following closed form solution of (25)

$$u(\xi) = \frac{\sqrt{209}}{722}\left[120 - sec^2\left(\frac{\lambda\xi}{2}\right)\left\{\left(15\cos\left(\frac{3\lambda\xi}{2}\right)sec\left(\frac{\lambda\xi}{2}\right) + 45\right) + i\left(15\sin\left(\frac{3\lambda\xi}{2}\right)sec\left(\frac{\lambda\xi}{2}\right) - 315tan\left(\frac{\lambda\xi}{2}\right)\right)\right\}\right] \quad (26)$$

where $\xi = (x - \omega t)$, and $\lambda = -\frac{\sqrt{209}}{19}$

Again choose $c_1 = -1$ and $c_2 = 1/\lambda$ the closed form solution (25) turns as

$$u(\xi) = \frac{\sqrt{209}}{722}\left[120 + cosec^2\left(\frac{\lambda\xi}{2}\right)\left\{\left(315 - 15\sin\left(\frac{3\lambda\xi}{2}\right)cosec\left(\frac{\lambda\xi}{2}\right)\right) + i\left(15\cos\left(\frac{3\lambda\xi}{2}\right)cosec\left(\frac{\lambda\xi}{2}\right) + 45cot\left(\frac{\lambda\xi}{2}\right)\right)\right\}\right] \quad (27)$$

4. Results and Discussion

In this section, we will discuss about the obtained solution of Kuramoto-Sivashinsky equation. Using the MSE method, achieve the traveling wave solutions from Eqs.(20) to (27). These solutions are general closed form traveling wave solutions which are soliton, and periodic wave solution respectively. From the above solution, the solutions (20) and (24) are represents in the exponential form where the solutions (21) to (23) and (25) to (27) are represents in terms of trigonometric functions. The solutions (22) and (26) are represents periodic wave solution and the solutions (23) and (27) are represents singular soliton solutions. Solutions (22) to (23) and (26) to (27) are in the complex form, so the modulus and arguments of these solutions have been plotted. The graph of modulus and the arguments of (22) are plotted in the figure-1 and figure-2 respectively for $a_0 = 0$, $a_1 = \frac{720}{19}$, $a_2 = \frac{180}{19}\sqrt{209}$, $a_3 = 120$, and $\omega = -\frac{30}{361}\sqrt{209}$ within the interval $-10 \le x, t \ge -10$. The graph of modulus of (27) is singular soliton is shown in the figure-3 for $a_0 = 2\omega$, $a_1 = \frac{720}{19}$, $a_2 = \frac{180}{19}\sqrt{209}$, $a_3 = 120$, and $\omega = \frac{30}{361}\sqrt{209}$ within the interval $-10 \le x, t \ge -10$. Others figures are omitted for convenience.

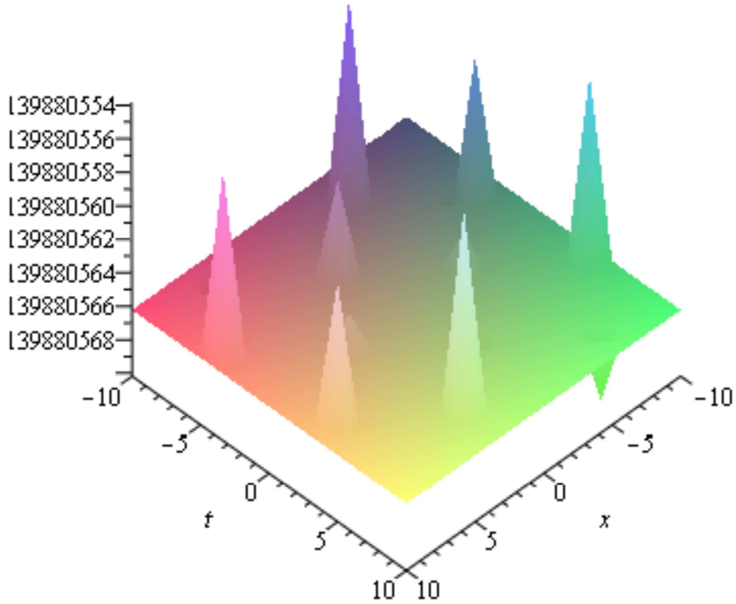

***Figure 1.** Plot of the modulus of solutions$u(\xi)$in (22) of Kuramoto-Sivashinsky equation.*

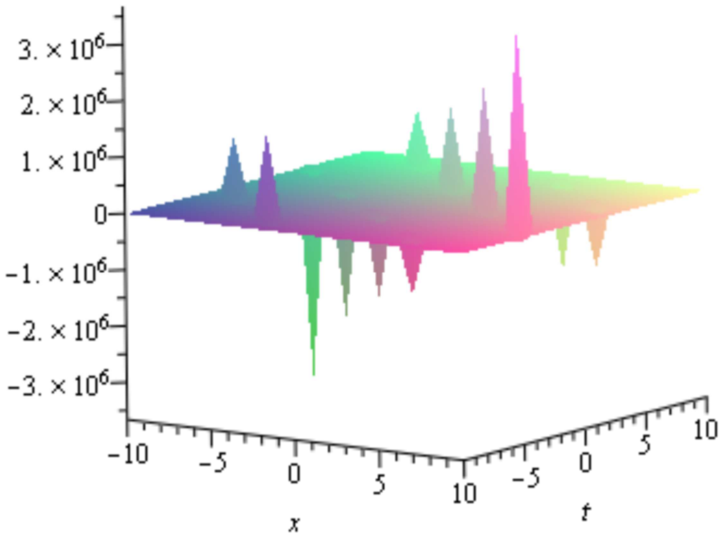

***Figure 2.** Plot of the argument of solutions $u(\xi)$in (22) of Kuramoto-Sivashinsky equation.*

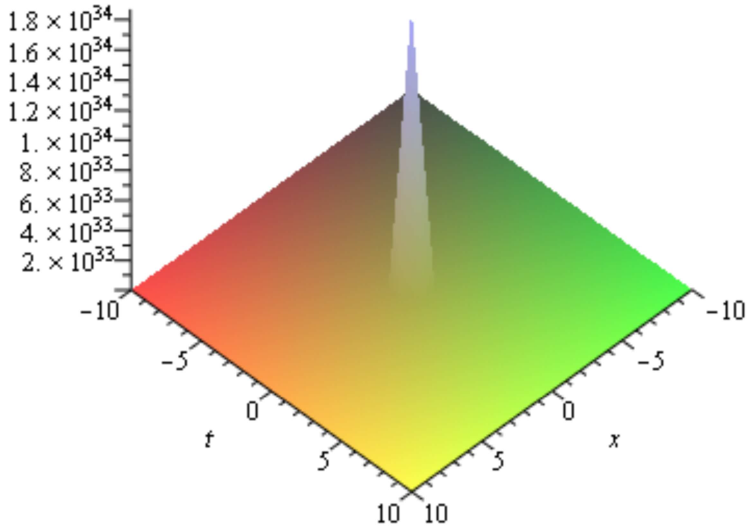

Figure 3. *Plot of the modulus of solutions* $u(\xi)$*in (27) of Kuramoto-Sivashinsky equation.*

5. Conclusion

In this article, the modified simple equation method has been successfully used to find the exact traveling wave solutions of Kuramoto-Sivashinsky equation. The solutions are verified to check the correctness of the solutions by putting them back into the original equation and found correct. It is significant to observe that, here, we obtained the value of the coefficients a_0, a_2 etc without using any symbolic computation software such as Maple, Mathematica, etc. Therefore this method is very easy and straightforward to handling. Also it is quite capable and can be applied for finding exact solutions of other NLEEs in mathematical physics.

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